



CP violating phase from minimal texture neutrino mass matrix: Test of the phase relevant to leptogenesis



Masataka Fukugita^{a,b}, Yuya Kaneta^{c,*}, Yusuke Shimizu^d, Morimitsu Tanimoto^e, Tsutomu T. Yanagida^a

^a Kavli Institute for the Physics and Mathematics of the Universe, University of Tokyo, Kashiwa 277-8583, Japan

^b Institute for Advanced Study, Princeton, NJ08540, USA

^c Graduate School of Science and Technology, Niigata University, Niigata 950-2181, Japan

^d Graduate School of Science, Hiroshima University, Higashi-Hiroshima, 739-8526, Japan

^e Department of Physics, Niigata University, Niigata 950-2181, Japan

ARTICLE INFO

Article history:

Received 23 September 2016

Received in revised form 11 November 2016

Accepted 14 November 2016

Available online 17 November 2016

Editor: J. Hisano

ABSTRACT

The model of neutrino mass matrix with minimal texture is now tightly constrained by experiment so that it can yield a prediction for the phase of CP violation. This phase is predicted to lie in the range $\delta_{CP} = 0.77\pi - 1.24\pi$. If neutrino oscillation experiment would find the CP violation phase outside this range, this means that the minimal-texture neutrino mass matrix, the element of which is all real, fails and the neutrino mass matrix must be complex, i.e., the phase must be present that is responsible for leptogenesis.

© 2016 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.

Following the discovery of neutrino oscillation, we proposed a neutrino mass matrix with the minimal texture (hereafter FTY model) [1] assuming that neutrinos are of the Majorana type, to understand a very large mixing between ν_μ and ν_τ found in atmospheric neutrinos. The 3×3 Dirac neutrino mass matrix has off-diagonal (1,2) and (2,3) elements in addition to one diagonal (3,3) element [2]. We assumed 3 degenerate right-handed neutrino masses for economy. By virtue of the seesaw mechanism [3,4] the mixing angle is a quartic root, rather than a square root [5], of the neutrino mass ratio and hence it can readily be large. This matrix was shown to give empirically determined mixing angles at a good accuracy [6]. There appeared much information as to neutrino mixing over the last two decades, but this matrix so far passed all critical passes. Notably, it predicted a finite mixing angle θ_{13} , which was established by now [7–9], and the exclusion of maximal mixing of θ_{23} [10].

Modern knowledge of the mixing angles allows an accurate and tight determination of the matrix element. The final, yet-to-be-known is the phase of CP violation, δ_{CP} .¹ A finite CP phase is being indicated in recent neutrino oscillation experiments [12,13]. The prime interest in this phase may be its possible role in lep-

togenesis [14]. We must first emphasize, however, that the phase that is visible in neutrino oscillation is not the phase that controls leptogenesis: δ_{CP} being finite does *not* mean leptogenesis. The phase that appears in neutrino oscillation arises from both neutrino and charged lepton sectors. A finite δ_{CP} may appear even if the neutrino mass matrix is real, and this is the case with the original FTY model. This, on the other hand, gives rise to the idea for an important test for the phase relevant to leptogenesis: whether the experimentally observed phase in neutrino oscillation deviates from the phase that is predicted from real matrices is a decisive test for a phase needed for leptogenesis.

In this paper, we predict the CP violating phase in our minimal texture of the neutrino mass matrix in light of new data of T2K [15] and NO ν A [10] experiments. We show that δ_{CP} is narrowly constrained with the presently available mixing data.

Our model [1,16,6], in the basis where the right-handed Majorana mass matrix is real diagonal, consists of mass matrices for the charged lepton and for the Dirac neutrino of the form [2],

$$m_\ell = \begin{pmatrix} 0 & A_\ell & 0 \\ A_\ell & 0 & B_\ell \\ 0 & B_\ell & C_\ell \end{pmatrix}, \quad m_{\nu D} = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & 0 & B_\nu \\ 0 & B_\nu & C_\nu \end{pmatrix}, \quad (1)$$

where each entry is complex in general: in our convention right-handed fermions operate from the left and left-handed from the right of the matrix. The phase of ν_R is fixed by this convention.

* Corresponding author.

E-mail address: kaneta@muse.sc.niigata-u.ac.jp (Y. Kaneta).

¹ For an earlier attempt, see ref. [11].

We have five phases in the neutrino mass matrix, but three of them can be absorbed into the wave functions of left-handed doublets and two were left, say those of B_ν and of C_ν in the third row. In the FTY model we have assumed these two to be real to make the problem analytically tractable, i.e., the elements of the neutrino mass matrix M_R and $m_{\nu D}$ are all real. The result determined thereof turns out to agree with neutrino experiment accurately [6] within the currently available accuracy. We retain this reality assumption in order to keep CP invariance in the neutrino sector. For the charged lepton mass matrix, three among the five phases are absorbed into the wave function of right-handed charged leptons, and hence two are left to us. This reality of the neutrino mass matrix is pivotal in the argument developed in this paper.

We start our analysis with the unit matrix for the right-handed Majorana neutrino as in the FTY model,

$$M_R = M_0 \mathbf{1}, \quad (2)$$

i.e., $M_{R1} = M_{R2} = M_{R3}$, but this assumption is *ad hoc* and will be relaxed to accommodate possible hierarchy in M_R . In this case the difference in masses (eigenvalues) can be absorbed into the wave functions of the right-handed neutrinos, which in turn leads to the violation of the symmetric (i.e., minimal) matrix structure of the Dirac neutrino mass. We later relax the assumption of degenerate M_R and introduce parameters,

$$K_{31} = M_{R3}/M_{R1}, \quad K_{32} = M_{R3}/M_{R2}, \quad (3)$$

to extend our model, avoiding the *ad hoc* assumption, but keeping reality of the matrix elements. We still take a diagonal basis for M_R .²

We remark that the neutrino mass is stable against radiative corrections. For the heavy right-handed neutrino of mass $O(10^{10})$ GeV the Yukawa coupling for the Dirac neutrino mass is smaller than $10^{-2.5}$. A calculation with the normalization group equation (e.g., [18]) gives the radiative correction of the order 10^{-6} relative to the leading term, which is negligible.

We obtain the three light neutrino masses, m_i ($i = 1, 2, 3$), as

$$m_i = \left(U_\nu^T m_{\nu D}^T M_R^{-1} m_{\nu D} U_\nu \right)_i. \quad (4)$$

With the real Dirac neutrino mass matrix the lepton mixing matrix is given by

$$U = U_\ell^\dagger Q U_\nu, \quad (5)$$

where the expressions of U_ℓ and U_ν , all their elements being real, are explicitly given in [16] in terms of the charged lepton mass and the neutrino mass, and Q is a phase matrix,

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & e^{i\tau} \end{pmatrix}, \quad (6)$$

which corresponds to the two phases left in the charged lepton mass matrix. Then, the CP violation in neutrino oscillations is written through phases σ and τ . For U we take the conventional 3×3 representation used by Particle Data Group.³

With the charged lepton masses, m_e, m_μ, m_τ , given, the number of parameters in our model is six, $m_{1D}, m_{2D}, m_{3D}, \sigma, \tau$ and M_0 , where M_0 is basically fixed by the neutrino mass, so that the number of parameters is five that are to be determined from empirical neutrino mixing angles. We note that this is the minimum

Table 1

Parameters of the neutrino mass matrix. The errors stand for 2σ .

Parameters	$K_{31} = K_{32} = 1$	$K_{31} \neq 1$ & $K_{32} \neq 1$
m_1 (meV)	$1.2_{-0.5}^{+0.6}$	0–6.6
m_2 (meV)	$8.7_{-0.2}^{+0.3}$	$8.7_{-0.3}^{+2.3}$
m_3 (meV)	49.5 ± 1.0	49.5 ± 1.5
$\sum m_i$ (meV)	59.5 ± 1.5	$58.5_{-1.5}^{+9.3}$
m_{ee} (meV)	$4.2_{-0.6}^{+0.8}$	$3.6_{-0.5}^{+5.2}$
$\sin^2 \theta_{23}$	$0.45_{-0.05}^{+0.02}$	$0.46_{-0.06}^{+0.10}$
δ_{CP} (radian)	$(0.89_{-0.12}^{+0.11})\pi, (1.11_{-0.11}^{+0.13})\pi$	$(0.83_{-0.09}^{+0.17})\pi, (1.17_{-0.17}^{+0.09})\pi$
K_{31}	1	0–1.3
K_{32}	1	0.3–1.5

texture of the 3×3 neutrino mass matrix, in the sense that reducing one more matrix element (i.e., letting A, B or C to zero) leads to the neutrino mixing that is in a gross disagreement with experiment. An antisymmetric mass matrix with 3 finite elements also leads to a gross disagreement, as one can readily see. So, our matrix is essentially necessary and sufficient, the unique form of the neutrino mass matrix under the requirement of minimum structure, i.e., four texture zeroes.

The lepton mixing matrix elements can be analytically computed, and are given approximately by the expression as written in Eq. (6) of Ref. [6]. It has been shown [16] that only normal neutrino mass hierarchy is allowed: the model does not accommodate inverted hierarchy nor degenerate neutrinos. It is also shown that $|U_{e3}|$ cannot be too small.

We obtain neutrino mass matrix elements, including the phase τ and σ , with Monte Carlo sampling in 5 parameter space. We adopt the data [19], taking 2σ as the limit:

$$\begin{aligned} \Delta m_{23}^2 &= (2.457 \pm 0.047) \times 10^{-3} \text{ eV}^2, \\ \Delta m_{12}^2 &= 7.50_{-0.17}^{+0.19} \times 10^{-5} \text{ eV}^2, \\ \sin^2 \theta_{12} &= 0.304_{-0.012}^{+0.013}, \quad \sin^2 \theta_{23} = 0.452_{-0.028}^{+0.052}, \\ \sin^2 \theta_{13} &= 0.0218 \pm 0.0010, \end{aligned} \quad (7)$$

where Δm_{23}^2 and Δm_{12}^2 represent mass difference squares relevant to atmospheric neutrino and solar neutrino experiments. We remark that all oscillation data are fit with our model well within one sigma errors of experiment. We take the lowest neutrino mass m_1 as a free parameter, while m_2 and m_3 are fixed by the mass differences.

All neutrino mass parameters are specified in our model, as given in Table 1, where errors shown are at 2σ . We also added the prediction for $\sin^2 \theta_{23}$, the experimental information for which still has a large errors and is not restrictive to the model. In the second column we assume $K_{31} = K_{32} = 1$, the original FTY model. We predict the effective mass that appears in neutrinoless double beta decay

$$m_{ee} = \left| \sum_{i=1}^3 m_i U_{ei}^2 \right|, \quad (8)$$

to lay in the range $m_{ee} = 3.6\text{--}5.0$ meV. The phase parameters σ and τ are well constrained to lie around $(\pi/2, 3\pi/2)$ or $(3\pi/2, \pi/2)$, where the former range disappears if $\delta_{CP} > \pi$. This will give the phase derived in neutrino oscillation, even if the neutrino mass matrix elements are all real.

We note that $\sin^2 \theta_{23}$ is constrained to lie in the range 0.40–0.47, where the upper limit comes from empirical θ_{13} , which is restricted to a narrow range by modern experiments. This means that maximal mixing, $\sin^2 \theta_{23} = 1/2$, is not allowed for θ_{23} , which agrees with the recent experiment [10].

² See ref. [17] for an attempt in a similar direction.

³ In case of the Majorana neutrinos unitary neutrino mixing matrix U_ν may be cast into the form $U_\nu = VP$, where P is a diagonal phase matrix with two Majorana phases. In our parameterization these phases are transferred into σ and τ .

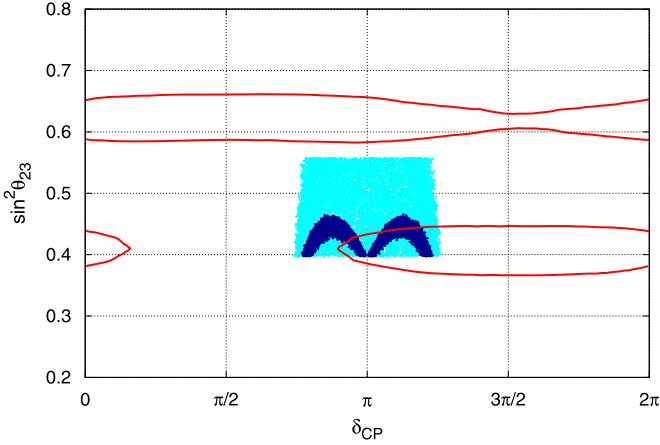


Fig. 1. Predicted δ_{CP} versus $\sin^2 \theta_{23}$. Dark blue (thick) region denotes the prediction when $K_{31} = K_{32} = 1$, and cyan (thin) shows the prediction for $K_{31} \neq 1$ and/or $K_{32} \neq 1$. All predictions are at 2σ . The regions inside red contours are allowed in NO ν A experiment at 1σ [10]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

We extend the model lifting the *ad hoc* assumption that K_{31} and K_{32} are equal to unity (see ref. [17]). We keep the reality of the matrix. We give in column 3 of Table 1, the neutrino mass parameters in this extended model. We find a limit on K_{31} that it must be smaller than 1.3, else we are led to too small a $\sin^2 \theta_{23}$ to be compatible with experiment. There is no lower limit for K_{31} . We see that $m_1 \propto K_{31}$, in so far as $m_1 \ll (\Delta m_{21}^2)^{1/2}$, so m_1 can vanish. We find that mixing angles vary little towards the limit $K_{31} \rightarrow 0$, i.e., the agreement is kept with experiment: only m_1 becomes small. We also find the limit $0.3 < K_{32} < 1.5$, the upper limit from the lower limit of θ_{23} , and the lower limit from θ_{13} to keep it not too small compared to experiment.

It may be appropriate to comment that the simple relation between mixing angles and mass ratios is lost in the extended model, when K_{31} is far from unity, as one can easily see for a two generation example. Namely, $m_1 = 0$ does not mean a vanishing relevant mixing angle. In fact, in our case we see that the mixing angles change only a little, as we take a limit $K_{31} \rightarrow 0$.

When we allow $K_{31} < 1$, m_{ee} may be smaller: for $K_{31} < 0.2$, m_{ee} takes 3–3.8 meV. On the other hand, m_{ee} can be as large as 9 meV for $K_{31} \approx 1/2$ and $K_{32} \approx 1/2$. This is the upper limit of m_{ee} attainable in our model. Here, m_1 gives a dominant contribution to m_{ee} .

Fig. 1 shows our prediction of δ_{CP} versus $\sin^2 \theta_{23}$ at 2σ .⁴ This CP phase is correlated significantly with $\sin^2 \theta_{23}$ than with other mixing angles. Thick symbols stand for our original $K_{31} = K_{32} = 1$ model, and thin symbols show the region allowed for $K_{31} \neq 1$ and $K_{32} \neq 1$. We also draw contours obtained by current NO ν A experiment at 1σ . A significant overlap is still seen between the prediction and experiment [10]. T2K experiment reported earlier favors a finite CP violating phase [12]. Their result is consistent with the newly reported NO ν A experiment. It is not shown in the figure, however, because their analysis assumes a fixed value, $\sin^2 \theta_{23} = 0.5$.

The extended model allows $\sin^2 \theta_{23}$ from 0.4 to 0.55, including maximal mixing 0.5. We see that the region allowed for δ_{CP}

extends only little upon the inclusion of free K_{31} and K_{32} parameters. If a more accurate experiment in the future would fall in the region outside the prediction, it compels that we must introduce a phase in the neutrino mass matrix. This serves as a decisive test of the current minimal texture model with neutrino mass matrix elements being real.

What is relevant to leptogenesis is the factor $[(m_{\nu D} m_{\nu D}^\dagger)_{i3}]^2$ to which the phase contributes. Whether lepton or antilepton excess is determined by sign of the phase of $[(m_{\nu D} m_{\nu D}^\dagger)_{i3}]^2$, and that of $M_i - M_3$. In our analysis, e.g., both $K_{31} > 1$ and < 1 are still allowed. The relation between the amount of lepton asymmetry and those phases, however, depends on details of the model beyond the mixing matrix, most importantly on the relative right-handed neutrino masses (i.e., the size of K_{ij}), which we cannot constrain from experiment at the current accuracy and our current knowledge. Therefore, we do not discuss leptogenesis further in the present paper.

When the oscillation data becomes more accurate, we may hope that the sign of the mass difference may eventually be predicted within the model without resorting to a new type of the experimental information.

While the agreement of the current model with recent precision experiment does not preclude the presence of the phase in the neutrino mass matrix, the disagreement, on the other hand, would compel us, in so far as we keep minimal texture, to introduce a phase. This is the phase that is needed to cause lepton asymmetry, or leptogenesis.

We conclude that our minimal texture model with real neutrino mass matrix, devised when neutrino oscillation was first reported, passed all tests concerning neutrino mixing that have been newly raised over two decades; the predictions progressively improved upon new experiments from time to time turned out to be so far all consistent with later experiments. It describes accurately the neutrino mixing parameters available today, and as a result it is now tightly constrained so that we can predict the CP violating phase. We proposed here, as the final test for the minimal neutrino mass matrix, a test whether predicted δ_{CP} agrees with experiment.

If it does, there is no compelling reason to introduce complex matrix elements in the neutrino mass matrix, i.e., no compelling reason that leptogenesis should occur in this model. If experiment would give δ_{CP} deviated from our prediction, we are led to have intrinsically complex matrix element that means a phase that is responsible for leptogenesis. Neutrinoless double beta decay, if it would give $m_{ee} > 9$ meV, also falsifies our minimal texture model.

Our analysis gives an example that the phase to be found in neutrino oscillation does not mean the phase that causes leptogenesis. This phase can be detected when real neutrino mass matrix fails to describe the phase derived from neutrino oscillation. More generally, this is an example of the strategy as to how to detect the phase intrinsic to the neutrino, requiring first the neutrino mass matrix to be strictly real and detecting if any deviation from it needed.

Acknowledgements

This work is supported by JSPS Grants-in-Aid for Scientific Research Nos. 15K05019 (MF), 16J05332 (YS), 15K05045, 16H00862 (MT) and 26287039, 26104009, 16H02176 (TTY). M.F. is supported by the Ambrose Monell Foundation in Princeton. This work receives a support at IPMU by World Premier International Research Center Initiative of the Ministry of Education in Japan.

⁴ One may calculate the two Majorana phases in the phase matrix $P = \text{diag}(1, \exp(i\alpha/2), \exp(i\beta/2))$, if U_ν would be cast into $U_\nu = VP$, from matrix elements together with σ and τ we obtained. For our original FTY model, thus obtained α is small but non-zero ($\sim \pm 13^\circ$). In the extended case $K \neq 0$ the solution includes $\alpha = 0$. β is poorly determined, including zero. We do not discuss these phases further, as they do not directly appear in experiment.

References

- [1] M. Fukugita, M. Tanimoto, T. Yanagida, *Prog. Theor. Phys.* 89 (1993) 263.
- [2] H. Fritzsch, *Phys. Lett. B* 73 (1978) 317, *Nucl. Phys. B* 115 (1979) 189.
- [3] P. Minkowski, *Phys. Lett. B* 67 (1977) 421.
- [4] T. Yanagida, in: *Workshop on the Unified Theory and Baryon Number in the Universe, 1979* (KEK report KEK-79-18); *Conf. Proc. C* 95 (1979) 7902131; M. Gell-Mann, P. Ramond, R. Slansky, in: P. van Nieuwenhuizen, D.Z. Freedman (Eds.), *Supergravity*, vol. 315, 1979; *Conf. Proc. C* 315 (1979) 790927.
- [5] S. Weinberg, *Trans. New York Acad. Sci.* 38 (1977) 185–201, HUTP-77-A057.
- [6] M. Fukugita, Y. Shimizu, M. Tanimoto, T.T. Yanagida, *Phys. Lett. B* 716 (2012) 294.
- [7] F.P. An, et al., Daya Bay Collaboration, *Phys. Rev. Lett.* 115 (2015) 111802.
- [8] S.B. Kim, et al., RENO Collaboration, *Nucl. Phys. B* 908 (2016) 94.
- [9] Y. Abe, et al., Double Chooz Collaboration, *J. High Energy Phys.* 1410 (2014) 086, Erratum: *J. High Energy Phys.* 1502 (2015) 074.
- [10] J. Bian, NOvA Collaboration, in: ICHEP2016, Chicago, 06 August, 2016.
- [11] S. Zhou, Z.z. Xing, *Eur. Phys. J. C* 38 (2005) 495, arXiv:hep-ph/0404188.
- [12] K. Abe, et al., T2K Collaboration, *Phys. Rev. Lett.* 112 (2014) 061802.
- [13] P. Adamson, et al., NOvA Collaboration, *Phys. Rev. Lett.* 116 (15) (2016) 151806.
- [14] M. Fukugita, T. Yanagida, *Phys. Lett. B* 174 (1986) 45.
- [15] K. Iwamoto, T2K Collaboration, in: ICHEP2016, Chicago, 06 August, 2016.
- [16] M. Fukugita, M. Tanimoto, T. Yanagida, *Phys. Lett. B* 562 (2003) 273.
- [17] M. Obara, Z.z. Xing, *Phys. Lett. B* 644 (2007) 136, arXiv:hep-ph/0608280.
- [18] C. Hagedorn, J. Kersten, M. Lindner, *Phys. Lett. B* 597 (2004) 63.
- [19] M.C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, *Nucl. Phys. B* 908 (2016) 199.